

Reply to “Comment on ‘Analytical and numerical verification of the Nernst theorem for metals’ ”

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In this reply to the preceding Comment [Klimchitskaya and Mostepanenko, Phys. Rev. E 77, 023101 (2008)], we summarize and maintain our position that the Drude dispersion relation when inserted in the Lifshitz formula gives a thermodynamically satisfactory description of the Casimir force, also in the limiting case when the relaxation frequency goes to zero (perfect crystals).

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The main concern of the authors of Ref. [1] is that in their opinion the Drude dielectric function is not valid if the metal is a perfect crystal, with no impurities at all. By contrast, if the metal is imperfect implying some relaxation, the authors seem to agree with our standpoint as expressed in Ref. [2]: The Drude dispersion relation is adequate to describe the medium and there is no conflict with the Nernst theorem in thermodynamics. The Drude relation implies that there is a finite relaxation frequency, which we call ν . We find it satisfactory that now there seems to be full agreement as far as imperfect crystals are concerned. The problem remains for perfect crystals only.

We can agree with the claim of Ref. [1] that perfect crystals is an idealization that forms much of the basis of solid state physics. From a physical viewpoint taking the limit $\nu \rightarrow 0$ is however in the Casimir context a delicate and not unique operation. There are several ways to argue that our results found via use of the Drude relation are valid in general.

(1) In practice, all solids have some relaxation. This follows also from the Kramers-Kronig relations, for any material having frequency dispersion. Ideal crystals can be seen as the limit of no relaxation $\nu \rightarrow 0$. We have shown very accurately, both analytically and numerically [2]—cf. also Refs. [3,4]—that for $\nu > 0$ the entropy $S=0$ at $T=0$. From this we draw the conclusion that in the limit $\nu \rightarrow 0$ the value $S=0$ is preserved. But the entropy decreases more and more rapidly when T increases from $T=0$, and this decrease becomes a discontinuous jump in the limit $\nu \rightarrow 0$. The variation of the entropy as a function of T for two copper half-spaces is nicely illustrated in Fig. 3 of the paper by Sernelius and Boström [5], for a finite value of ν .

(2) Further, merely taking the limit $\nu \rightarrow 0$ is not fully satisfactory from a physical viewpoint. It should be noted that

on the molecular level the permittivity becomes nonlocal, $\epsilon = \epsilon(\omega, k)$, where k is the magnitude of the wave vector \mathbf{k} . Then ϵ becomes finite for finite \mathbf{k} . Only the special case $\epsilon(0, 0)$ is infinite, and this (“measure zero”) cannot be expected to give a finite contribution to the Casimir force. In independent studies Svetovoy and Esquivel [6] and Sernelius [7] find that the transverse electric (TE) zero mode should not contribute when spatial dispersion is carefully taken into account. Because of nonlocal effects, the former of these state, “the question does [ν] go to zero or have some residual value at $T \rightarrow 0$ becomes unimportant” and they find that the TE zero frequency contribution must indeed be zero to satisfy Nernst’s theorem.

(3) A TE zero frequency mode is not a solution of Maxwell’s equations and should not occur for that reason. This point is discussed in more detail in Refs. [4,8]. (See especially Sec. III of Ref. [8].)

(4) Introducing a TE zero frequency mode for ideal crystals would imply that a medium with $\nu=0$ would behave differently from a real medium when taking the limit $\nu \rightarrow 0$. In the former case with the TE mode included the entropy increases slowly from $S=0$ when T increases, while in the latter case it decreases rapidly to negative values for a real medium with $\nu > 0$. Thus, in the limit $\nu \rightarrow 0$ the result of the latter is different from the former. Due to this the use of the latter for $\nu \rightarrow 0$ and the former for $\nu=0$ would create a discontinuity in physical properties at $\nu=0$ that we find unphysical.

In connection with Eq. (3) in Ref. [1] it is argued that $\nu(T) \ll \zeta_m(T) = 2\pi kmT/\hbar$ ($m=1, 2, 3, \dots$) holds as temperature decreases. Surely, this is so for a perfect lattice in which relaxation follows the Bloch-Grüneisen formula all the way to zero temperature. We consider real metals with impurities, however, in which ν reaches some residual value at low temperatures. At temperatures so low that relaxation is dominated by impurities, their Eq. (3) no longer holds, but for sufficiently small T , Eq. (1) does. The authors of [1] explain in great detail why, for this reason, our calculated low temperature correction is numerically inaccurate. This is true,

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but immaterial, since our analysis is phenomenological and does not aspire to provide exact numerical estimates. The numerical calculation presented was motivated by a wish to confirm the correctness of our analytical result, which it convincingly does.

The question becomes how to define the perfect lattice at zero temperature since the limits $\nu \rightarrow 0$ and $T \rightarrow 0$ do not commute. One may set $\nu=0$ right at the outset, or take the limit of vanishing relaxation frequency subsequent to calculating the free energy. For reasons discussed above we suggest the latter procedure is the more appropriate. If Eq. (3) of [1] is to hold at all temperatures, the former procedure must be employed, which results in a different prediction of the Casimir force and fulfillment of Nernst's theorem by inclusion of the TE zero frequency mode.

Let us sum up: The basic assumption underlying our calculation in Ref. [2] was that the relaxation frequency ν stays finite at any temperature including $T=0$. At low temperatures we expect ν to be smaller than at room temperature; this changes our results quantitatively but not qualitatively. On the basis of this assumption, we showed that there is no thermodynamic inconsistency associated with the Drude relation. Now, simply setting $\nu=0$ makes our formalism to some extent indeterminate. In particular, this indetermination persists if $\nu=0$ for $T=0$. Physical arguments, such as the requirement of continuity of behavior with respect to different values of input parameters, suggest strongly, however, that the results obtained from the Drude relation are quite general.

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- [1] G. L. Klimchitskaya and V. M. Mostepanenko, preceding Comment, Phys. Rev. E **77**, 023101 (2008).
- [2] J. S. Høye, I. Brevik, S. A. Ellingsen, and J. B. Aarseth, Phys. Rev. E **75**, 051127 (2007).
- [3] I. Brevik, S. A. Ellingsen, J. S. Høye, and K. A. Milton, J. Phys. A (to be published), e-print arXiv:0710.4882.
- [4] J. S. Høye, I. Brevik, J. B. Aarseth, and K. A. Milton, Phys. Rev. E **67**, 056116 (2003).
- [5] Bo E. Sernelius and M. Boström, in *Quantum Field Theory under the Influence of External Conditions*, edited by K. A. Milton (Rinton, Princeton, NJ, 2004), p. 82.
- [6] V. B. Svetovoy and R. Esquivel, Phys. Rev. E **72**, 036113 (2005).
- [7] Bo E. Sernelius, Phys. Rev. B **71**, 235114 (2005).
- [8] J. S. Høye, I. Brevik, and J. B. Aarseth, Phys. Rev. E **63**, 051101 (2001).